MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

05 MARCH 2020
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5)** printed pages inclusive of the cover page and formulae sheet.
2. Answer **ALL** four questions in the answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [20 marks]

a) Andy plays a game against Danny according to the following rules: Each of them starts with $10. From a deck of 52 playing cards, a card is randomly drawn with replacement for each game. If a card of Jack, Queen, King or Ace is drawn, then Andy wins $1 from the Danny, else the Danny wins $1 from Andy. The game continues until either player has no more money.

   i) What is the probability that the gambler wins? [6 marks]
   ii) What will be the expected duration of the game? [4 marks]

b) Given that cars passing the Sunway toll with the rate of 3 per minute starting from 7am.
   i) Find the probability that it takes less than 1.5 minutes to have 3 cars passing the toll. [7 marks]
   ii) Find the probability that time between 2 cars passing the toll is at least 2 minutes. [3 marks]

Question 2 [20 marks]

a) At an outpatient mental health clinic, appointment cancellations occur at the rate of 1.5 per day.
   i) What is the probability that one cancellation on a particular Wednesday? [2 marks]
   ii) What is the probability that at least 3 cancellations a week given that the clinic is closed during weekend? [10 marks]

b) At one hotel in KL, the time spent by customers waiting for an elevator follows a uniform distribution between 0 and 3.5 minutes.
   i) Find the probability that a customer waits less than 2 minutes. [4 marks]
   ii) Find the probability that a customer waits more than 150 seconds. [4 marks]

Continued...

Question 3 [30 marks]

a) Given a random variable X has the following probability mass function:

$$f(x)=\begin{cases}\frac{1}{4} & ; \quad x=10,15,20,25\\ 0 & ; \quad otherwise\end{cases}$$

i) Find mean and E (X^2). [5 marks]
ii) Find the moment generating function of X. [4 marks]
iii) Verify the mean value using moment generating function. [5 marks]

b) Let the joint pdf of (X,Y) be given by

$$f(x,y)=25e^{-5y} \qquad \text{for} \quad 0<x<0.2 \text{ and } y>0.$$

i) Find the marginal pdf for X and Y. [10 marks]
ii) Find E(XY). [6 marks]

Question 4 [30 marks]

Consider a Markov chain with state $\{1, 2\}$ and transition probability matrix

$$P=\begin{bmatrix}0.45 & 0.55\\ 0.2 & 0.8\end{bmatrix}$$

a) Draw the transition diagram. [5 marks]
b) Determine for every state whether it is absorbing. [5 marks]
c) Determine for each state whether it is transient or persistent. [20 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution $P(X = x) = p^x q^{1-x}$ for $x = 0, 1$
Binomial Probability Distribution $P(X = x) = {}^nC_x\, p^x q^{n-x}$ for $x = 0, 1, \ldots, n$
Poisson Probability Distribution $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$
Geometric Probability Distribution $P(X = x) = pq^{x-1}$ for $x = 1, 2, \ldots$
Uniform Probability Distribution $f(x) = \dfrac{1}{b-a}$ for $a < x < b$
Exponential Probability Distribution $f(x) = \lambda e^{-\lambda x}$ for $x > 0$

B. MOMENT

kth Moment about the origin of X: $\mu'_k = E[X^k]$
Moment Generating Function for X: $M_X(t) = E\left(e^{tx}\right)$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)}$$ where m and n are both even or both odd.

$E(X_n) = n(p - q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{\left(\frac{q}{p}\right)^a - \left(\frac{q}{p}\right)^c}{1-\left(\frac{q}{p}\right)^c} & , \quad p \neq q \\ 1-\dfrac{a}{c} & , \quad p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = \begin{cases} \dfrac{1}{q-p}\left\{a - c\left[\dfrac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\} & , \quad p \neq q \\ a(c-a) & , \quad p = q \end{cases}$$

D. POISSON PROCESS

Pmf for N(t) :

$$P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

Pdf for T_n:

$$f(t) = \lambda e^{-\lambda t}$$

Pdf for S_n:

$$f(t) = \frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$